Towards Understanding Jovian Planet Migration

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Abstract

We present 2D hydrodynamic simulations of circumstellar disks around protostars using a 'Piecewise Parabolic Method' (PPM) code. We include a point mass embedded within the disk and follow the migration of that point mass through the disk. Companions with masses $M_c \gtrsim 0.5 M_J$ can open a gap in the disk sufficient to halt rapid migration through the disk. Lower mass companions open gaps, but migration continues because sufficient disk mass remains close to the disk to exert large tidal torques. We find that the torques which dominate the migration of low mass planets originate within a radial region within 1–2 Hill radii of the planet's orbit radius, a distance smaller than the thickness of the disk. We conclude that a very high resolution 3D treatment will be required to adequately describe the planet's migration.

1 Initial conditions

The PPM code and initial conditions are very similar to those presented in Nelson *et al.* 1998. We begin with a one $M_\odot$ protostar fixed to the origin of our coordinate system. We assume that a disk of mass $M_D = 0.05 M_\odot$ is contained between the inner and outer grid boundaries at 0.5 AU and 20 AU and that the disk is self gravitating. A second point mass (the 'planet') is set in a circular orbit at a radius 5.2 AU away from the protostar and is free to migrate through the disk in response to gravitational forces. No other forces act on the planet and it does not accrete mass from the disk. In different simulations, we investigate migration rates of different planet masses.

The disk mass is distributed on a 128×224 cylindrical (r, ϕ) grid with a surface density given by a power law, $\Sigma(r) = \Sigma_1 (1AU/r)^p$, where Σ_1 is determined from the assumed disk mass and $p = 3/2$. We assume an initial temperature profile with a similar power law, $T(r) = T_1 (1AU/r)^q$, where the temperature at 1 AU is $T_1 = 250$ K and $q = 1/2$. These initial conditions produce a radial profile for which the minimum Toomre Q (of ~ 5) is found near the outer disk edge. The profile exhibits a steep increase in the inner regions due to the increased effects of pressure on the orbital characteristics there. A single component isothermal gas equation of state is used to derive pressure at each point in the disk.

Velocities are determined assuming initial rotational equilibrium. Radial velocities throughout are assumed equal to zero, while angular velocities are determined by balancing the gravitational, pressure and centrifugal forces in the disk.

2 Migration rates with varying planet mass

Using the initial conditions outlined above we have completed a series of simulations, varying the planet mass assumed for each simulation. We show the effect of a $1 M_J$ mass planet on the disk in figure 1. Within a few hundred years, the planet raises very large amplitude spiral structures which lead (trail) the planet radially inside (outside) its orbit radius and cause a gap to form around the planet. These structures are similar to those in previous work (Bryden *et al.* 1999, Kley 1999) where the planet's trajectory was fixed to a single orbit radius.

The orbital trajectory of the planet (figure 2) is strongly affected by the gravitational torques from the spiral structures. We show the After a $\sim 50-100$ yr period in which the planet first builds spiral structures, the migration rate is constant for the next 500 yr, then drops to zero and the planet reaches a 'final' orbit radius of about 3.5 AU, after 800 yr. The migration rates fit for simulations with different disk masses are shown in figure 3, and are valid for the period for which the migration rate is constant (see figure 2), i.e. the period for which the migration can be considered 'Type I'. The fitted migration rates are very rapid: about 1 AU per thousand years, so that migration in to the stellar surface would occur before the end of the disk lifetime of 10^6 yr. Still higher mass planets move so quickly through the disk that they 'outrun' their own gap formation efforts and fall into the star. With more realistic initial conditions (a $2 M_J$ planet should already have a gap), we expect this phenomenon to go away.

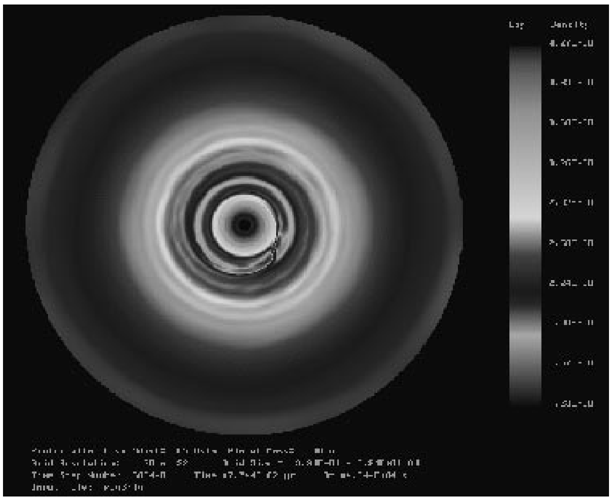

Figure 1: The surface density of the disk after 775 yr of evolution. As shown by the deep blue region in the plot, the planet has built a deep gap in the disk, but some spiral structure remains present in the gap. The planet has only just stopped its rapid migration through the disk at this time.

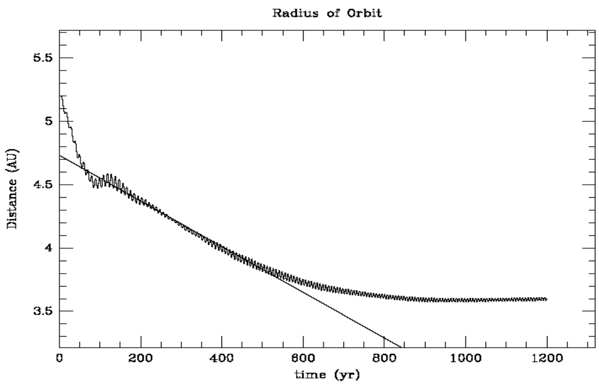

Figure 2: The orbital radius of the $1 M_J$ planet shown in figure 1 as a function of time. The planet begins at 5.2 AU and migrates inward to ~ 3.5 AU, before its inward trajectory is affected by the formation of a sufficiently deep and wide gap.

Over the course of the first several hundred years of evolution, planets with mass less than $2 M_J$ hollow out a deep gap in the disk, which extends all the way around the star. In figure 4 we show the azimuth averaged surface density structure at several points during the evolution of the system shown above. The gap forms quickly after the beginning of the simulation and within 500 years has hollowed out a region about 3 AU wide. The surface density near the planet (200–300 gm/cm^2) is a factor ten below the initial profile and less (100 gm/cm^2) at its deepest, just outside the planet's orbit. By the end of the simulation, the gap has deepened to a factor of 100 less than its unperturbed profile and continued to get deeper even at the end of our simulations. It does not substantially increase its width after initial formation however.

The existence of a gap eventually causes the migration to slow and, if it becomes deep and wide enough, decrease to a rate defined by the viscosity of the disk ('Type II' migration) rather than to dynamical processes like gravitational torques ('Type I' migration). From figure 4 we can determine the approximate disk conditions which define the transition between Type I and Type II migration. From figure 2 we see that the

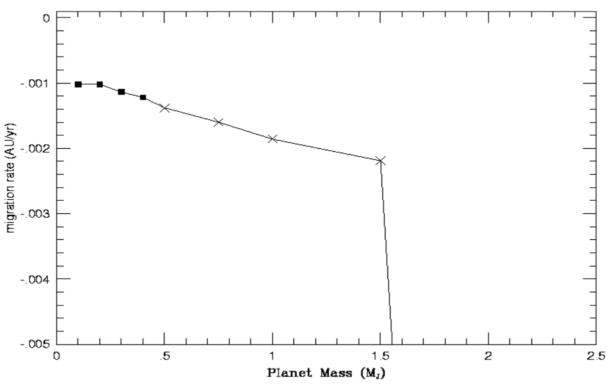

Figure 3: Migration rates of planets of varying mass. Points marked '×' are able to evacuate a gap large enough and deep enough to halt their migration. Planets greater than $1.5M_J$ 'outrun' their own gap formation efforts and rapidly migrate inward to the inner grid boundary.

migration rate decreases starting after about 5-600 yr of evolution. Comparison with figure 4 shows that the onset of the transition occurs when the gap is 3 AU wide and has surface density $\Sigma_{gap} \sim 200-300$gm/cm^2. The transition is concluded and further rapid orbital decay is suppressed by the time the system has evolved for 700 yr, when the surface density is $\Sigma_{gap} \sim 100$ gm/cm^2.

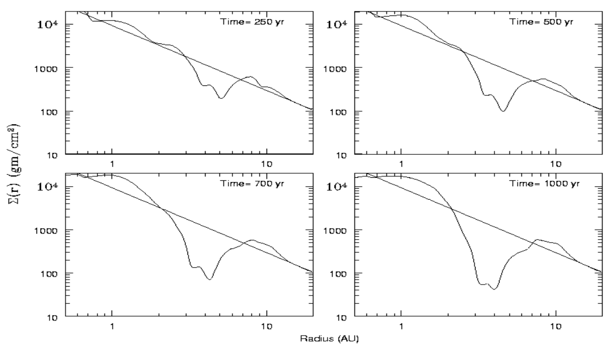

Figure 4: Azimuth averaged density structure at several times during the evolution.

The values of surface density and gap width required for the onset and completion of the transition to Type II migration are typical of of each of the simulations we have performed, withouth regard to planet mass. Simulations with 0.3–0.5M_J planets are able to enter the transition to Type II migration but over the duration of our simulations (1800 yr) do not complete the transition. We continue to evolve these simulations further in time to determine their ultimate fate.

3 The relative importance of the disk close to the planet

Linear theory (e.g. Takeuchi *et al.* 1996) predicts that the most important Fourier components of the spiral patterns raised in the disk will be those with azimuthal wavenumber m=20–40, corresponding to an azimuthal wavelength near the planet of about 1 AU. The Lindblad resonances of these patterns will be at a distance radially inward and outward from the planet of $R_{LR} = a(1 \pm 1/m)^{2/3} \approx 0.3$ AU, where a is the semi-major axis of the planet. Another relevant parameter is the Hill radius, $R_H = a(M_J/3M_*)^{1/3} \approx 0.3$ AU, defining the sphere of influence of the planet. Further, the z structure of the disk becomes important on the same spatial scales because in 2D the gravity will be effectively 'amplified' by the assumption that all the disk matter is in the $z = 0$ plane rather than some at high altitudes more distant from the planet. Unfortunately each of these values are similar to the grid resolution size scale of computationally affordable simulations.

In order to characterize both physical and numerical effects we have performed a series of simulations in which we vary the effective gravitational softening length of the planet, effectively 'turning on' or off the effect of matter very close to the planet. The migration rates obtained from such a series of simulations are shown in figure 5. We again assume the initial conditions as above, with a planet of mass $0.3M_J$.

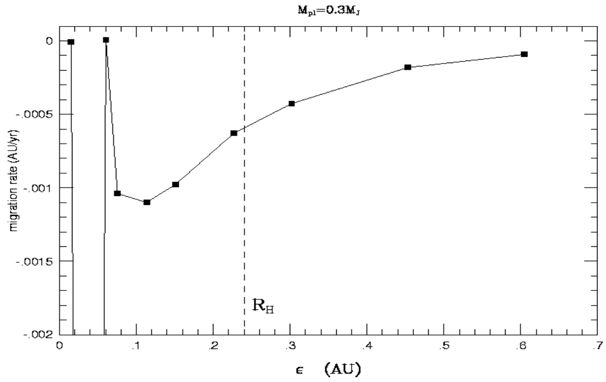

Figure 5: The migration rate of a $0.3M_J$ planet with varying assumed gravitational softening length. The planet's Hill radius is shown with a vertical dashed line. From the plot it is clear that the most influenetial portion of the disk for migration is within 1–2 Hill radii of the planet.

The migration rate increases by a factor of about five as the softening decreases from 0.5 AU to 0.1 AU, clearly showing the importance of the distribution of disk matter close to the planet. The largest increases occur as ϵ decreases to the size of the Hill radius or smaller. Below $\epsilon = 0.08$ AU, or half the size of one grid cell, the migration rate is numerically unstable and slight changes in the softening change the migration rate from zero to 1 AU/100 yr (below the bottom of the plot). The dependence of the migration on the disk matter very near the planet is in qualitative agreement with with the conclusion of Ward (1997), who showed that the most important contribution to the migration will come from the disk matter within about one disk scale height (in our case $\sim$ 0.3 AU near the planet) of the planet's radial position. These results show that very high resolution three dimensional simulations of the region around the planet will be required in order to understand the migration rates of a planet through the disk.

4 Summary

In the course of this study we have

• Shown that planets evolved in a 2D disk without a gap migrate inward through a substantial fraction of their initial semi-major axis radius on timescales $\sim$ 1000 yr.

• Shown that planets with masses higher than $\sim 0.5M_J$ can open a gap sufficiently wide and deep to drastically slow their migration through the disk (i.e. transition to Type II migration). Conditions for beginning the transition are that surface density of the disk near the planet be about 200-200 gm/cm^2 and that the gap be $\sim$ 3 AU wide. Conditions for completing the transition to Type II migration are surface densities near the disk of $\sim$ 100 gm/cm^2.

• Demostrated the critical importance of very high spatial resolution of the disk near the planet required for correct evolution of the planet's migration.

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